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Detection of a cylindrical boundary diffraction wave emanating from a straight edge by light interaction

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ABSTRACT

The paper shows that the boundary diffraction wave originating at an edge is an omnidirectional cylindrical wave. The experimental set-up used to demonstrate this property employs a He-Ne laser beam. The beam is split into three beams using a glass plate. One of the beams passes straight through, the second beam passes through the glass plate and the third beam is the reflected beam. It is shown that the interference patterns are observed in all three beams. Analysis of these patterns shows that the boundary diffraction wave originating from the edge is an omnidirectional cylindrical wave. This analysis also provides strong evidence that the boundary diffraction wave travels not only within the beam where it originates but also to the neighboring coherent beam. The energy re-distribution was also shown to be dependent on the wavelength of the incident light beam and hence provides further evidence as to why longer wave lengths disperse more compared to shorter wavelengths in white light diffraction by an edge.

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1. Introduction

Some of nature's stunning phenomena, seen by human eyes, are created by light diffraction and it could be said that optics as one of the primary fields that led to the development of modern science. Italian scientist Grimaldi (1613–1663) was the first to describe the observation of diffraction [1] scientifically and termed the effect as “diffraction” which was derived by the Latin word “diffringere”, meaning “to break into pieces”.

Later, the essential features of the diffraction phenomena were explained by Christiaan Huygen (1629–1695). The Huygen's principle [2] states that the propagation of a light wave can be predicted by assuming that each point of the wave front acts as a source of secondary wavelets. The envelop of all these secondary waves, is the new wave front. The mathematical consideration of Huygens's principle is given by the Fresnel–Kirchhoff formula [3,4] which is derived fundamentally by Green's theorem and the regular wave equation with introduction of some basic simplifying assumptions.

In contrast to the Fresnel–Kirchhoff integral formula, the boundary diffraction wave (BDW) model does not count points at the aperture as a source to construct the resultant intensity at the observer point but uses the direct contribution from the source and the each point in the boundary edge (see Fig. 1). Maggi

[5] and Rubinowicz [6] both showed that the diffraction pattern at a point P can be obtained by superposition of two waves, one from direct wave, $U_g(P)$ and the other originating from the edge, $U_d(P)$ as in the expression [7]

$$U(P) = U_g(P) + U_d(P) \quad (1)$$

where

$$U_g(P) = \frac{\exp(jkr)}{r} \quad (2)$$

and

$$U_d(P) = -\frac{1}{4\pi} \int_c \frac{\exp[-jk(r_0 + r_b)] \sin(r_0, dl)}{r_0 r_b \sin(\alpha/2)} dl \quad (3)$$

U_g (Eq. (2)) is the geometrical contribution which includes incident, reflected and directly transmitted waves from the light source S (see Fig. 1) and U_d (Eq. (3))—line integral along the edge contour of C) is the boundary diffraction wave contribution originating from the knife edge K of the diffracting object to the point P where observation is being made. The resultant $U(P)$ is then the superposition of two components $U_g(P)$ and $U_d(P)$, where r is the direct distance from light source S to the point of observation P, r_0 is the distance between S and K, r_b is the distance between K and point P and QUOTE α is the angle between SK and KP.

$U_g = \exp(jkr)/r$ shows the amplitude and phase at P when exposed to the geometrical wave. $U_g(P) = 0$ when P is a point where there is no contribution from the geometrical wave. $U_d(P)$ is the key component that produces the diffraction phenomenon.

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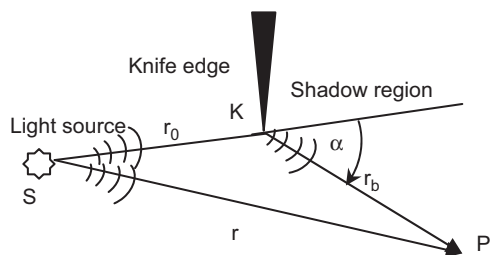


Fig. 1. Schematic diagram of the hypothesis of boundary diffraction wave. S is the source of light, K is the knife edge, and P is the observation point, where r is the direct distance from light source, S , to the point of observation, P , r_0 is the distance between S and knife edge, r_b is the distance between knife edge and point P , α is the angle between SK and KP .

However at the shadow boundary where α equals zero, the line integral approaches infinity and a discontinuity occurs. Therefore a uniform theory of BDW and its mathematical and physical properties have emerged and are lengthily discussed in the recent years. In the previous publication by the author [19], an experimental evidence for the existence of the BDW and its propagation across a light beam have been shown. The experiment [19] also showed that the propagation of boundary diffraction wave clearly created the wave shapes “near” and “far” from the slit, which are identical to the well known Fresnel and Fraunhofer diffraction patterns, respectively. This paper is intended to provide further evidence to strengthen the concept of BDW and its propagation nature. The author primarily discusses the observations with fundamental concepts (the wave, electromagnetic and superposition theories) by providing semi-quantitative discussions in order to visualize the cylindrical nature of BDW.

Although the generation of a boundary wave in the presence of a physical body which interacted with a light beam was first introduced by Young [9,10] in 1802, there was no thorough discussion of the boundary wave, unlike the Fresnel–Kirchhoff model of diffraction, until recent times. Also most of the documentation in optics is based on Huygens’s definition and its mathematical interpretations. However, fair numbers of publications [11–18] related to the BDW and its mathematical and physical properties have emerged and are lengthily discussed in the recent years. In the previous publication by the author [19], an experimental evidence for the existence of the BDW and its propagation across a light beam have been shown. The experiment [19] also showed that the propagation of boundary diffraction wave clearly created the wave shapes “near” and “far” from the slit, which are identical to the well known Fresnel and Fraunhofer diffraction patterns, respectively. This paper is intended to provide further evidence to strengthen the concept of BDW and its propagation nature. The author primarily discusses the observations with fundamental concepts (the wave, electromagnetic and superposition theories) by providing semi-quantitative discussions in order to visualize the cylindrical nature of BDW.

2. Experiment

The experimental set-up is presented in Fig. 2. A He-Ne laser beam (Throlab HRR050, wavelength 632.8 nm) was used as the primary light source. The experiment was performed in two stages. In stage one, the monochromatic light beam was interrupted by a glass plate to produce three beams, B_1 (part of the incident beam), B_2 (refracted beam through glass slide) and B_3 (reflected beam from the surface of the glass slide). Splitting of the beams by the angled glass plate is depicted in Fig. 2. One end of each beam originates at the edge of the glass shown in Fig. 5. The intensity profiles of the beams were then recorded. A movable line camera (Throlab, LC1-USB, 3000 pixels in 24.5 mm length, 7 $\mu\text{m}/\text{pixel}$), equipped with a linear charged coupled device (CCD), to measure intensity profile of beams B_1 and B_2 , was placed behind the glass edge at a distance d_b . Similarly the CCD array was placed in front of the glass slide for B_3 at a distance d_f .

For each line measurement of a specific cross section in the beam, the energy profiles obtained were identical but in different widths and intensities. All measurements were taken at the center of the beam, where the intensity and width were the maximum. The specific details of this experimental set-up have been discussed elsewhere [19].

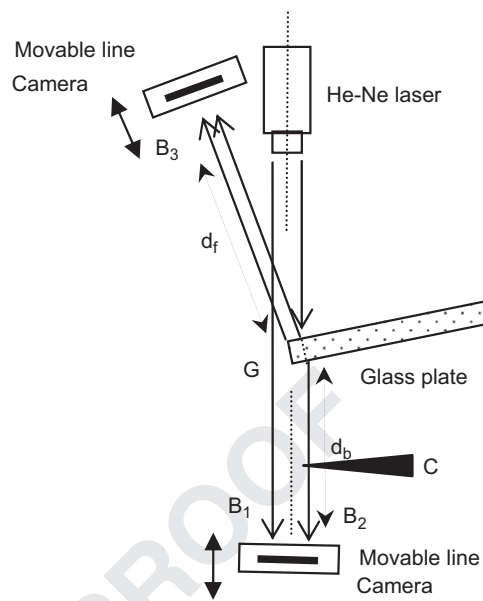


Fig. 2. Experimental arrangement. A laser beam (Throlab HRR050.1 He-Ne laser, 632.8) is divided into three coherent beams, B_1 , B_2 and B_3 using a glass plate. Part of the incident beam B_1 and beam refracted through the glass slide B_2 propagates in the forward direction while reflected beam B_3 propagates in the backward direction. A movable line camera, equipped with a linear charged coupled device (CCD) as a light sensor (Throlab, LC1-USB, 3000 pixels in 24.5 mm length, 7 μm pixel) is placed behind the glass edge and records the intensity profile in the plane of CCD sensor. Similarly the CCD array moves to front side of the glass slide in order to record the beam profile of B_3 . Distances to the CCD from the glass plate are d_b and d_f in the backside and front side of the glass plate respectively.

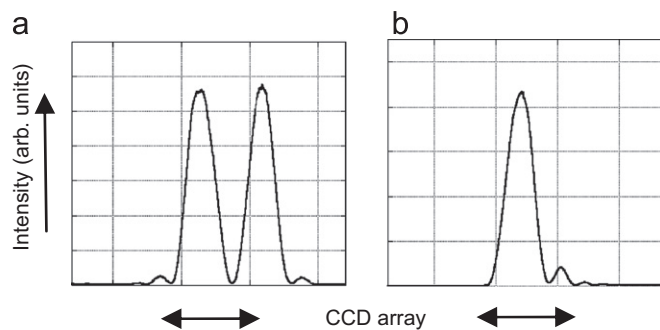


Fig. 3. Intensity profiles of beams recorded at a distance of 700 mm from the glass edge. (a) Intensity profile of B_1 and B_2 . (b) Intensity profile of B_3 . Vertical axis shows intensity in arbitrary units. Horizontal axis represents the plane of CCD array.

Two beams, B_1 and B_2 , occur at the air-glass interface but the wide separation is due to the interference caused by the path difference of two beams through two different media, air and glass respectively. In the second stage, a knife edge is introduced behind the glass plate (Fig. 2). The intensity profiles of B_1 and B_2 were then recorded from the knife edge to 1500 mm in 20 increments (see Fig. 2) in order to study the BDW originating at the knife edge C and traveling across B_2 and B_1 respectively. The second experiment was designed not only to study boundary diffraction wave itself but also to verify that the BDW (disturbance) travels not only within the beam where it originates but also outside (in this case, beam B_1). Two measurements (intensity profiles) for each distance from knife edge, with and without the knife edge were made. The magnitude of the “change of intensity” of the original wave due to BDW was obtained by subtracting the intensity distribution of original wave from the intensity distribution of the disturbed wave due to the knife edge. Observations at five distances (0 mm, 10 mm, 100 mm, 500 mm, and 1500 mm)

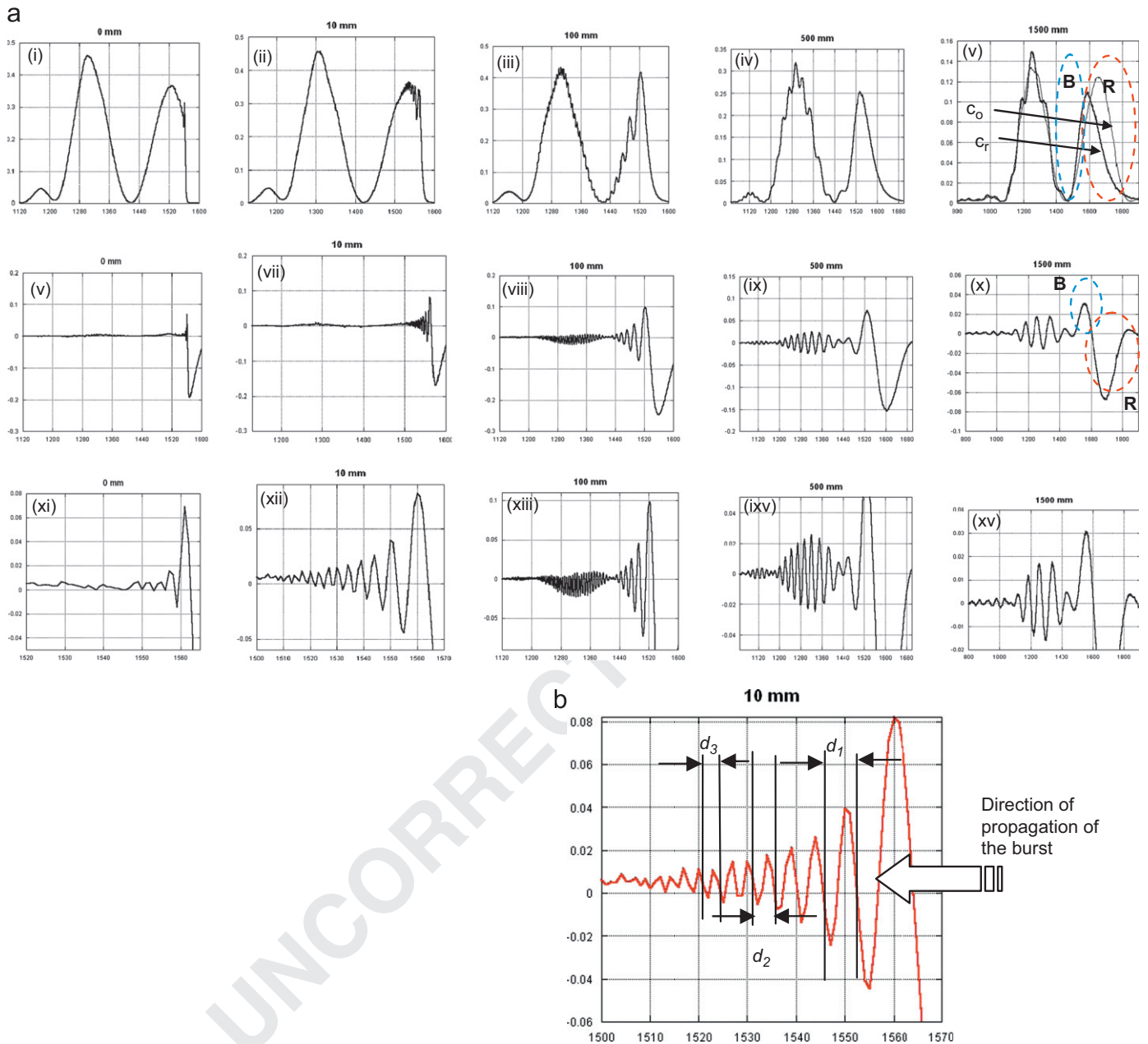


Fig. 4. Intensity profiles recorded by the line camera at different distances from the knife edge, C . a(i)–a(v) depict the disturbed intensity profiles at distances 0 mm, 10 mm, 100 mm, 500 mm and 1000 mm. a(vi)–a(x) show the filtered change of intensity of B_1 and B_2 due to interference caused by the boundary wave. Vertical and horizontal scales were kept unchanged; (xi)–a(xv) show enlarged view of a(v)–a(x). Both vertical and horizontal scales were expanded in order to show details. (b) Profile of the change of intensity of B_2 due to the interference caused by the boundary wave at a distance of 10 mm from the knife edge. Distances between maxima become shorter at the left side, $d_3 < d_2 < d_1$. The disturbance moves energy of the original distribution (curve C_0 in (v)) from right to left, forming a new energy re-distribution (curve C_r in (v)). The amount of energy removed from the area R is denoted by a red dotted circle and extra energy gained in area B is denoted by a blue dotted circle in Fig. 5(x). It is assumed that intensity is directly proportional to the energy of the beam.

are presented for convenience. Disturbed waves Fig. 4(i)–(v) and their filtered intensity changes (Fig. 4(vi)–(x)) are depicted in the same intensity scale. Fig. 4(xi) and (xv) are enlarged views of Fig. 4(vi) and (x) respectively for clarity.

3. Observations

The observations made of the triple beams, part of the incident and two originated at the glass edge, are shown in Fig. 3. Fig. 3(a) and (b) shows the intensity distributions of beams B_1 and B_2 as well as B_3 respectively at a distance of 700 mm from the

glass edge. It is clear that all three beams B_1 , B_2 and B_3 behave similarly, suggesting that the boundary wave originated at the glass edge travels across a beam by rearranging the intensity distribution (in other words energy distribution) of the main beam as shown in Fig. 5.

Fig. 4 shows that the traveling disturbance due to the knife edge C crosses B_2 and then to B_1 . The disturbance shows a change in intensity (as a burst) in the observation plane of the beams B_1 and B_2 . This “intensity change” enlarges itself while moving alone with beams. It was also noted that the rate of change of intensity of the burst is higher in the front of the bust (left side of Fig. 4(b)).

4. Discussion

The three beams B_1 , B_2 and B_3 originate from the common glass edge G. All these beams, shown in Fig. 5, undergo influence of a secondary wave, the “boundary diffraction wave”, emanating from the glass edge G. The patterns of the beams B_1 , B_2 , and B_3 showed similar intensity profiles (Fig. 3(a) and (b)) at a distance of 700 mm. It seems that the disturbances that occur at the boundary travel across the beam and altered the intensity profile (or energy profile) of three beams B_1 , B_2 , and B_3 . Shifted energy is shown in circled areas, x, y, and z in Fig. 5. The Comparison of the re-distributed energies of the beams with and without the glass edge is shown in Fig. 5(b).

The data obtained in the second part of the experiment was used to interpret the observations that were made in the first part of the experiment. Similar to the glass edge, G, a disturbance occurred at the knife edge C in beam B_2 , which altered the intensity profile (re-distribute energy). To verify that B_1 and B_2 were independent,

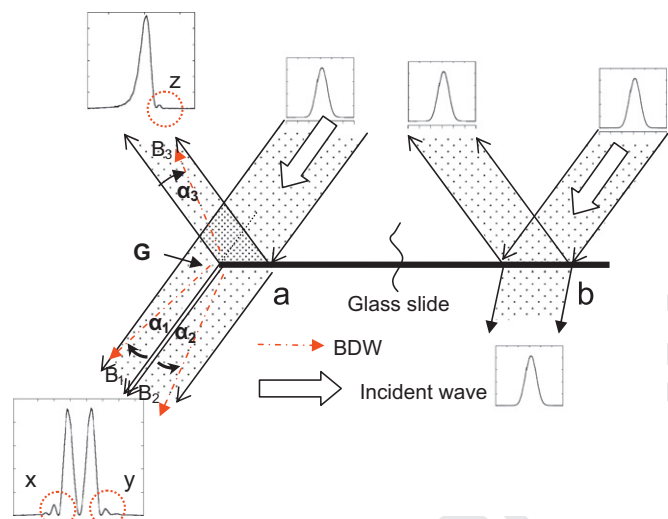


Fig. 5. Splitting of light beam into three parts by a glass plate. (a) A beam incident on an edge of a glass slide produces three beams B_1 , B_2 and B_3 . The intensity profiles at a distance of 700 mm of three light beams show a intensity patterns, X,Y and Z at a side which is opposite to the side where glass edge is situated. (b) Light beam incident on a plane surface of a glass plate lacking an edge creates refracted and reflected beams. The intensity profile of these beams has similar Gaussian distribution as that of the incident beam.

the individual beams were blocked. This showed that the intensity profile of the remaining beam was unaffected.

The measurements of intensity profiles along the beams B_1 and B_2 (Fig. 4) provide evidence on how this disturbance (“burst” or boundary diffraction wave) travels by altering the energy profiles of beams [19]. The change in disturbance travels from beam B_2 to beam B_1 . See Fig. 4. At first, the disturbance occurred only in B_2 (Fig. 4(i) and (ii)) and then moved to B_1 as in Fig. 4 (iii)–(v). Therefore this data provides strong evidence that the disturbance (BDW) travels not only within the beam where it originates but also neighboring coherent beam (in this case, beam B_1). The recorded intensity pattern is similar to the interference occurring with two coherent beams. It is important to note that the “rate of change of intensity per unit distance” of the interference pattern decreases in the burst horizontally towards the boundary edge. This can be measured by the gradually decreasing distances d_1 , d_2 and d_3 between the intensity maxima of the burst (see Fig. 4(b)). Interference for two plane waves produces equally distanced peaks [20] and hence this observation cannot be considered as plane wave interference.

The beam emitted from the He-Ne laser in the fundamental mode has a perfect plane wave front and hence B_1 – B_3 are also plane waves. Therefore a plausible explanation for the changes of distances between intensity maxima could be that the plane wave emitted from the laser interferes with circular or cylindrical waves (shown in Fig. 6).

Fig. 6(a) graphically depicts a plane wave interfering with a cylindrical wave emanating from its side which is analogous to the BDW created at the glass edge G or metal edge C (Fig. 2). Plane wave XY meets cylindrical wave at a_0, a_2, \dots, a_i to produce maxima. When moving along Y to X, the angle of the tangent AB to the plane wave XY (α) increases. Ergo this gives rise to gradually reduced distances between consecutive maxima [19] as seen in experimental observation in Fig. 4s(b) and 6(c).

As stated before, the resultant intensity in the burst is higher at the right side in Fig. 4(xi) and (xii). This could be partially explained with the model of the BDW presented in Fig. 6(a) by the increase of radius of the BDW which will lead to a reduction of energy density of the cylindrical wave and thus to the intensity profiles (Fig. 4(xi) and(xii)). The resultant intensity (I) in interference for two waves with maximum intensities I_1, I_2 and phases ϕ_1, ϕ_2 is given by.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\phi_1 - \phi_2) \tag{4}$$

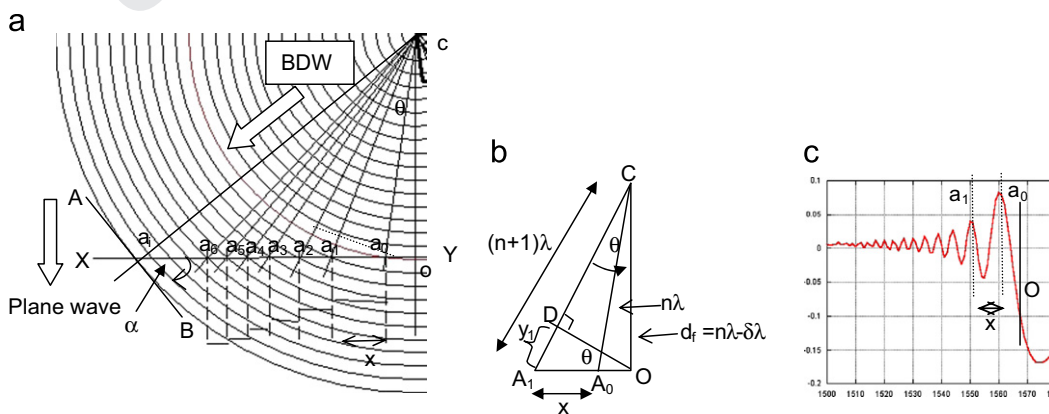


Fig. 6. Interference of a coherent plane and a cylindrical wave. (a) Two coherent planes and cylindrical waves produce energy re-distribution by interference. XY represents the plane wave and AB is the tangent at a_i to the cylindrical wave originating at c. α is the angle between the AB and XY at a_i . Intensity maxima occur at a_1, a_2, \dots, a_i . co is the distance between knife edge c and the CCD detector. x is the distance between intensity peaks a_0 and a_1 . (b) Geometrical drawing of the area coa_1 in Fig. 7(a). $A_0 = a_0, A_1 = a_1, O = o, a_0$ and a_1 represent the 1st and 2nd peaks in the intensity distribution of the diffracted wave, respectively. Length of CA_0 is $n\lambda$ and CA_1 in $(n+1)\lambda$. The distance CO is slightly shorter than CA_0 by $\delta\lambda$. θ is the angle A_1CO . y is the distance from A_1 to the point D which is the point of intersection of the perpendicular line to A_1C , drawn from point O. (c) Filtered intensity distribution from a situation similar to that of Fig. 7(a) when $CO = 10$ mm.

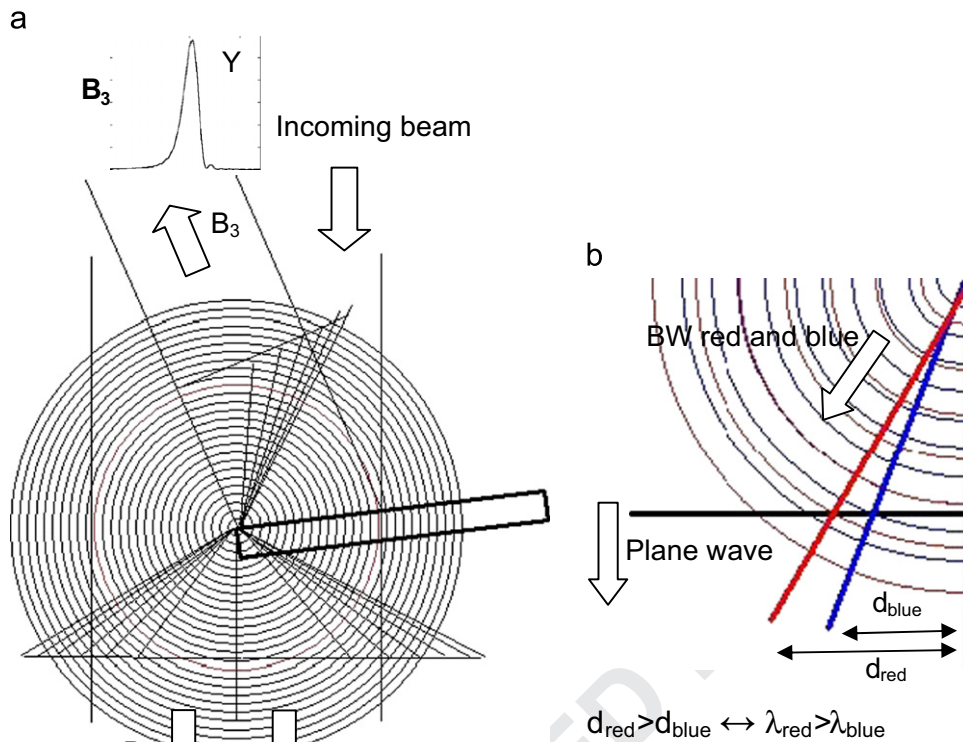


Fig. 7. Summarizes the observation about the glass edge and explains the dispersion of white light at an edge. (a) Cylindrical wave initiated at the glass boundary edge propagates omnidirectionally and interferes with B1, B2 and B3. The interference causes re-distribution of energy in B1, B2, and B3 and leads to curves X and Y. (b) Red has longer wavelength than blue; and therefore red color disperses more than blue color. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

For the plane wave, maximum intensity stays nearly constant (let us say I_1 in Eq. 4) while for a cylindrical wave intensity (I_2) reduces with its radius. The phase difference ($\phi_1 - \phi_2$) oscillates between $-\pi$ and $+\pi$ when cylindrical waves cross the plane wave XY and it reflects in intensity fluctuations (intensity between maxima and minima) as seen in Fig. 4.

It also seems that the magnitude of the intensity fluctuation between maxima and minima of peaks increases along the direction of propagation of plane wave. Therefore, it appears that this pattern, of higher intensity variation of the peaks, is altered, when the peak intensity reaches a certain threshold, to fit the intensity profile of the main beam (Fig. 4(xiii)-(xv)). Change of I_2 causes this pattern of the resultant I , the pattern of the burst, but increasing magnitude between minima and maxima (burst intensity) along the direction of propagation of the plane wave is unanswered by the classical Equation (4). To increase this variation, intensity I_1 has to decrease or intensity I_2 has to increase according to Eq. (4). In reality both intensities I_1 and I_2 decrease in the direction of propagation. The intensity of I_2 is weaker than I_1 and during propagation I_2 continues to weaken (relative to I_1 , because BDW originates only from relatively very small fraction of energy of parent beam) and according to Eq. (4) the resultant variation in I should decrease along the direction of propagation. However as discussed before, the opposite is observed in this paper; the "variation of I " increases during propagation. This will be discussed in a separate article.

There is also a resolution limit of the LC1-USB CCD camera. However this limit applies only to the distance between intensity variations smaller than CCD pixel width ($7 \mu\text{m}$). The portion of the data considered contains distances between intensity variations that are larger than the pixel size.

The region coa_1 in Fig. 6(a) is depicted as COA_1 in Fig. 6(b) (A_1 as a_1 , A_2 as a_2). $a_1 a_2$ is the distance between 1st two peaks and can be measured by the line camera data. CO is the physical distance from the knife edge C and the plane of the line camera sensor, XY. CA_0 is the distance to the first peak a_0 in fig. 6(a) and CA_1 is the distance to the second peak from the knife edge C. If the length of CA_0 is $n\lambda$, where n is an integer then CA_1 should occur at the next wavelength distance (Fig. 6(a)) and equal to $(n+1)\lambda$. CO is perpendicular to the plane XY and therefore the length CO should be less than CA_0 . The difference in distance is a certain fraction of a wavelength, $\delta\lambda$. Experimental value of the $\delta\lambda$ is $\sim \lambda/2$ at CO equal to 10 mm (distance between a_0 and a_1 represents phase difference of 2π or λ). Substituting the distances when $CO = 10$ mm to the triangle COA_1 it is possible to show that the $A_1 D$ (where D is obtained from the line drawn perpendicular to CA_1 from point O), y_1 , is approximately 3λ for large n values by approximating $(n+0.5)$ and $(n+1)n$. The relation ($y_1 \approx 3\lambda$) holds also for other values of CO. The relation can be extended to

$$3\lambda \approx y = \sin(\tan^{-1} \theta) \quad (5)$$

where $\theta = A_1 O / CO$.

Applying experimental values of A_1O , to respective distances, with the assumption $\delta\lambda \approx \lambda/2$, at $CO=10$ mm, λ approaches a value similar to the wavelength of the laser ($\lambda=632.8$) which is the wavelength of He-Ne used for the experiment. This shows that the cylindrical wave emanating from an axis coinciding with the boundary edge C will interfere with a plane progressive beam.

By using the cylindrical wave emanating from the glass edge it is possible to understand the experimental observation in Fig. 3. Fig. 7 introduces the cylindrical BDW model to the observations shown in Fig. 5. A cylindrical wave originating from the glass edge G travels outwards as shown in Fig. 6(a) while redistributing the energies of three beams by interference. The intensity of the beam is directly proportional to the energy [20] and therefore the intensity distribution can also be considered as energy distribution. The disturbance also (Fig. 4(v)) moves energy of the original distribution (curve c_o in Fig. 4(v)) from right to left, forming a new energy distribution (curve c_r in Fig. 4(v)). The amount of energy removed from the area R is denoted by a red dotted circle in Fig. 4(x). The energy shift in interference is discussed in details elsewhere [20,21].

This model which proposes a cylindrical BDW provides an alternative explanation for the color dispersion in white light diffraction. The increase of wavelength λ increases θ (Eq. (5)) and thereby increases the distance between maxima and hence larger dispersion in longer wavelengths. Therefore this model may also explain the large dispersion of red light compared to the blue light in white light diffraction.

5. Conclusions

In this experiment it is shown that a secondary wave known as boundary diffraction wave was emanated from a glass edge which interfered with the three light beams B_1 , B_2 , and B_3 . The intensity profiles of the interference were explained by proposing a cylindrical wave model for the boundary wave. The interference of a cylindrical wave with a plane wave re-distributes the energy of primary wave forming the classical diffraction pattern. This

data provides strong evidence that the disturbance (BDW) travels not only within the beam where it originates but also neighbouring beam. It is known that the energy at the boundary edge is only a small fraction of total energy of the beam; however this small fraction of energy causes a relatively significant shift in energy of the primary incident beam. Further investigations are required to explain this observation.

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References

- [1] F.M. Grimaldi, Bologna ("Bonomia"), Vittorio Bonati, Italy, 1665, pp 1-11.
- [2] C. Huygens, Traite de la Lumiere, Van der Aa, Leiden, 1690.
- [3] A. Fresnel, Oeuvres Completes d'Augustin Fresnel, 1, Imprimerie Imperiale, Paris, 1866, Tome.
- [4] H. Hecht, Optics, second ed., Addison Wesley, ISBN 020111609X, 1987, Chapters 5 and 6.
- [5] G.A. Maggi, Annals of Mathematics 16 (1888) 21.
- [6] A. Rubinowicz, Annals of Physics 53 (1917) 257.
- [7] Y.Z. Umul, Optics and Laser Technology 41 (2009) 285.
- [8] Y.Z. Umul, Journal of the Optical Society of America A 27 (2010) 1613.
- [9] T. Young, Philosophical Transactions of Royal Society of London 20 (1802) 12.
- [10] T. Young, Philosophical Transactions of Royal Society of London 92 (1804) 1.
- [11] G. Kirchhoff, Wied. Ann. 18 (1883) 663.
- [12] R.M. Lewis, J. Boersma, Journal of Mathematics and Physics 10 (1969) 2291.
- [13] R.G. Kouyoumjian, P.H. Pathak, Proceedings of the IEEE 62 (1974) 1448.
- [14] P. Liu, B. Lü, Optik (Stuttgart) 116 (2005) 449.
- [15] S. Ganci, American Journal of Physics 57 (1989) 370.
- [16] S. Ganci, European Journal of Physics 18 (1997) 229.
- [17] R. Kumar, Applied Physics B 90 (2008) 379.
- [18] P. Saari, P. Bowlan, H. Valtna-Lukner, M. Lõhmus, P. Piksarv, R. Trebino, Optics Express 18 (2010) 11083.
- [19] C.K.G. Piyadasa, European Physical Journal D 64 (2011) 505.
- [20] C.K.G. Piyadasa, Sri Lankan Journal of Physics 6 (2005) 51.
- [21] C.K.G. Piyadasa, Optik, doi:10.1016/j.jleo.2011.09.026, in press.